MODELLING THE GEOMETRIC RELATIONSHIP BETWEEN 2-D AND 3-D CHIP CURL IN MACHINING

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ABSTRACT

The works of Nakayama *et al* are representative of the prevailing view regarding how the classical 2D notions of up-curl and side-curl can be related to the geometry of helical (3D) chips generally obtained in practical machining operations. Recently, these views were re-examined by the authors and it was realised that the traditional definition and compounding of these radii is ambiguous. As a result a new geometric analysis of lightly obstructed chips was developed. The new analysis revealed that Nakayama's analysis with regard to up-curl was questionable. This led to a new equation for up-curl. The present paper summarizes the new analysis and its implications. It is shown that the entire geometry of the chip-in-process can be determined from four simple measurements conducted on the chip-in-hand. Finally, a new map of different chip forms that arise at different combinations of up-curl, side-curl and chip flow angle is presented.

1. INTRODUCTION

In the analysis of any mechanical phenomenon, the first issues to be addressed usually relate to the geometry of the problem. If our understanding of the geometry is inaccurate or incomplete, the subsequent analyses (e.g. force analysis) will be correspondingly in error.

In view of its importance in facilitating unmanned machining, chip breaking has emerged as a major field of study in recent years. It has long been recognised that chips are *born curled* (although when exactly a chip could be considered to have been *born* continues to be fuzzy) and that initially continuous chips usually break owing to encounters with external obstacles. The nature of these encounters clearly depends upon the form and path of the chip prior to the encounter. This brings us back to the question of chip geometry.

Most practical chips are 3-D in nature. However, as with any other aspect of chip formation, early literature on metal cutting was dominated by 2-D notions. In particular, two such notions gained widespread recognition: up-curl (of radius ρ_u) and side-curl (of radius ρ_s). Subsequently, a third parameter called the chip flow angle, η_c, was added.

Prevailing notions concerning how one may relate 3-D chip-curl to the 2-D notions of ρ_u, ρ_s, and η_c in the 3-D era are dominated by the views of Nakayama et al.

The first succinct presentation of this view was made in 1974 [2]. The same view was essentially reaffirmed fourteen years later in 1992 [2], which indicates the durability of these notions. Nakayama had developed his analysis in the context of cutting tools with plane rake faces. However, many of the insights derived from his work are applicable to tools with chip former features (intuitively designed bumps and grooves on the tool rake face).

Recently, while investigating some methods for controlling chip forms, the authors had occasion to revisit Nakayama's analysis [1, 2] and found the following notions underlying Nakayama's work to be very useful:

(i) The tool-chip separation line (TCSL) is *assumed* to be a straight line.

(ii) Upon leaving the TCSL, a 3-D continuous chip undergoes rigid body motion along a circular-helical path.

(iii) The form and path of the chip immediately after leaving the TCSL are essentially determined by the velocity distribution in the chip at the moment of its passing over the TCSL.

Observation (i) indicates that the world of chip formation meets the world of chip form at the TCSL. Hence it should in principle be possible to obtain much insight into the chip formation process by studying the geometric form of the chip output from the chip formation zone, i.e. from *chips-in-hand*. A review of metal cutting literature shows that only three properties of chips-in-hand have been exploited hitherto: the thickness, length, and width of the chip. However, there are many other properties of 3D chips-in-hand (e.g. outer radius ρ_0, inner radius ρ_1, pitch p, angle θ between the chips axis and the rake face, the direction of procesion of the chip, etc.) that, in principle, could be utilised to gain further insights into the chip formation zone itself.

Chips-in-hand (i.e. chips that have totally exited from the working zone) can be classified into two types: (i) *lightly obstructed* chips, and (ii) *strongly obstructed* chips.

Lightly obstructed chips are those that have not undergone significant plastic deformation after exiting from the TCSL. This occurs when the additional loading arising from the obstruction, if any, could be accommodated by a corresponding change in the stress pattern within the chip formation zone (shear zone plus the tool-chip contact zone).

Strongly obstructed chips occur when the additional loading could not be accommodated within the chip formation zone and, hence, there is additional plastic deformation (or, even, breaking) of the chip.

The present paper mainly addresses issues concerning lightly obstructed chips. In such situations, the geometry of the chip-in-hand directly represents the chip geometry at the moment the chip had passed over the TCSL.

In order to understand how a chip-in-hand could be utilised to gain insight into 3-D chip formation, the present authors conducted a rigorous analysis of the geometry of helical chips as they exited from the TCSL [3, 4]. However, rather unexpectedly, the analysis revealed several inconsistencies in Nakayama's analysis. The most significant of these related to the fact that Nakayama had taken up-curl to be occurring in a plane normal to the TCSL as well as the tool rake plane, whereas our analysis indicated that up-curl should be measured in a plane containing the chip velocity vector at the TCSL while being normal to the tool rake plane.

The main intention of the present paper is to highlight and resolve the inconsistencies associated with Nakayama's analysis [1, 2] and broadly discuss some of the implications of the revised analysis.

Generally, in the analysis of any 3-D object, one either starts from a given set of 2-D perceptions and tries to obtain the corresponding 3-D perception, or one starts from the 3-D perception to derive specific 2-D notions. Usually, the 3D perception is unique whereas it is possible to obtain an infinity of 2-D descriptions corresponding to the 3-D perception. In other words, the 3D→2D route is of the '*one-to-many'* type whereas the 2D→3-D route is of the *'many-to-one'* type and hence could lead to ambiguities.

Nakayama's analysis of chip-curl had followed the 2D→3-D route whereas the present paper adopts the 3D→2D route.

2. NAKAYAMA'S 2D→3-D ANALYSIS OF CHIP CURL

Nakayama et al had sought to determine the 3D geometry of a generalized helical chip by *"compounding"* the 2D notions of up-curl and side curl. In particular they sought to expressions several 3D characteristics (such as helix radius ρ, and pitch p) in terms of ρ_u, ρ_s, and η.

Figure 1b is an adaptation of the illustration used by Nakayama and Arai [2] in defining the radii of up-curl (ρ_u) and side curl (ρ_s) while machining with a tool with a plane rake face. In this illustration, axis X is along the TCSL, axis Y is perpendicular to X while being parallel to the rake plane, and axis Z is perpendicular to both X and Y (i.e. perpendicular to the rake plane). The origin is set on axis X at the end cutting edge side of the chip. According to [2], "when the two circular arcs in the Figure 2b are compounded, the helix in Figure 2a is produced". The radius of the arc in plane YZ is then taken as the radius of up-curl, ρ_u, whereas the radius of the arc in plane XY is taken as the radius of side-curl, ρ_s.

The following characteristics of the *pure forms* of up-curl and side-curl are recognized in [1, 2]:

***Pure* up-curl:** The TCSL is parallel to the cutting edge i.e. $\Delta\psi = 0$ where $\Delta\psi$ is the angle between the cutting edge and the TCSL. Hence the plane normal to the TCSL is identical to that normal to the cutting edge. Tool-chip contact length, l_c, chip velocity, $\mathbf{V}_c$, and ρ_u are uniform along the TCSL The chip axis is parallel to the rake plane, i.e. $\theta = 0$. Likewise, $\eta = 0$ and $\eta_c = 0$ (note that η_c is the conventional chip flow angle specified with reference to the cutting edge whereas η is specified with reference to the TCSL). The chip geometry is completely determined by considering just the plane normal to the cutting edge.

***Pure* side-curl:** V_c is linearly varying along the TCSL so that $\Delta\psi \neq 0$ and ρ_s is not constant along the TCSL. The chip axis is normal to the rake plane, i.e. $|\theta| = 90^o$. The chip geometry is completely determined by considering just the tool rake plane (i.e. the plane XY).

Nakayama *et al.* [1, 2] considered 3-D chip formation and *assumed* that plane YZ is the plane of up-curl and that the projections of velocity $\mathbf{V}_c$ of the chip particle on planes YZ and XY respectively match, as instantaneous linear velocities of rotations, the angular velocities of rotations with respect to up-curl and side-curl. Thus, interestingly, his analysis of chip curl adopts a *rotational* viewpoint whereas the traditional notions of up-curl and side-curl had been notions obtained from a *curvature* viewpoint.

Table 1. Equations developed or implied by Nakayama et al. [1, 2]

Eq. No.	Note	Equation
N1	See [4]	$\omega_z = V_c / \rho_s$
N2	See [4]	$\omega_x = (V_c \cos\eta) / \rho_u$
N3	See [5]	$\tan\theta = \omega_z / \omega_x = \rho_u / (\rho_s \cos\eta)$
N4	See [5]	$\rho = \sqrt{\dfrac{1 - \sin^2\eta\cos^2\theta}{(\cos\eta/\rho_u)^2 + (1/\rho_s)^2}}$
N5	By combining N3 and N4	$\rho_u = \dfrac{\rho\cos\eta}{\cos\theta\sqrt{1 - \sin^2\eta\cos^2\theta}}$
N6	By combining N3 and N4	$\rho_s = \dfrac{\rho}{\sin\theta\sqrt{1 - \sin^2\eta\cos^2\theta}}$

Nakayama et al noted that the angular velocity, $\boldsymbol{\omega}$, of the 3-D chip has only two non-zero components, $\boldsymbol{\omega}_x$ and $\boldsymbol{\omega}_z$, perpendicular to planes YZ and XY respectively. Hence they identified these as the angular velocities of up-curl in plane YZ and side-curl in plane XY respectively. Thus, the radii of up-curl and side-curl of 3-D chip were taken to be the radii of rotation in planes YZ and XY respectively. This procedure led to equations (N1) and (N2) for the rotational velocity components $\boldsymbol{\omega}_z$ and $\boldsymbol{\omega}_x$. These expressions were then utilized in relating to 3-D helical chip geometry to arrive at equations (N3) and (N4) for θ and ρ respectively. Note that, although Nakayama *et al.* [4, 5] had not explicitly stated them, a combination and rearrangement of equations (N3) and (N4) leads to equations (N5) and (N6) for ρ_u and ρ_s respectively (see Table 1).

3. A NEW 3D→2D ANALYSIS OF CHIP CURL

We have summarized the chip curl analysis due to Nakayama et al [1, 2] in the previous section. Note that this analysis had adopted (i) the 2D→3D route (they had first defined the 2-D notions of up-curl and side-curl and then compounded them to arrive at a perception of the 3D chip form), and (ii) a rotational viewpoint (see equations N1 to N3 in Table 1).

In contrast, we will now adopt (i) the 3D→2D route, and (ii) a curvature viewpoint (since, after all, the classical notions of up-curl and side-curl are basically notions of curvature and the corresponding rotational viewpoint is only incidental). It is worth noting that, when the present authors had started on this 3D→2D route, our intention was merely to verify the findings of Nakayama et al from a different perspective. However, as we will show below, we found (to our surprise) that Nakayama's equation N5 for ρ_u is questionable although his equation N6 for ρ_s is correct.

Figure 2 illustrates our geometrical analysis. It is assumed that the tool rake surface is plane (plane P_r). Let O_0OO_1 be the tool chip separation line (TCSL). Unlike Nakayama [1, 2], who had started with the assumption that the TCSL is a straight line, we will let the TCSL be a plane curve. Let O be an arbitrary point on the TCSL, and O_0 and O_1 the end points of the TCSL. The outer surface of the chip (henceforth called the *chip face*) can be considered to have been generated by the helical motion of the TCSL about axis $A_{\mathbf{H}}$. Points O_0 and O_1 generate circular helices $\mathbf{H}_0$ (of radius ρ_0) and $\mathbf{H}_1$ (of radius ρ_1) respectively. The convention adopted is that $\rho_0 \geq \rho_1$.

We will initially analyze the geometry of the helical chip face with reference to a right-handed Cartesian system, XYZ, centered at point O. Axis Z is normal to the rake plane, P_r, with the positive direction outward from the tool rake face (as in [1, 2]). Axis Y is normal to the projection, $A_{\mathbf{H}r}$, of the chip helix axis on P_r.

$\mathbf{V}_c$ is the velocity of the chip particle at O in a direction parallel to P_r [1]. Let η be the angle between $\mathbf{V}_c$ and axis Y. The trajectory of the chip particle starting at point O is a circular helix, $\mathbf{H}$. Hence $\mathbf{V}_c$ can be resolved into two orthogonal instantaneous velocities $\mathbf{V}_T$ and $\mathbf{V}_R$: $\mathbf{V}_T$ is the velocity of translation parallel to the helix axis, $A_{\mathbf{H}}$, and $\mathbf{V}_R$ is the rotational velocity corresponding to the angular velocity of rotation, $\boldsymbol{\omega}$, about $A_{\mathbf{H}}$. A steady state chip implies a TCSL that remains constant in space and time.

Let $O_{\mathbf{H}}$ be the point on $A_{\mathbf{H}}$ such that line $OO_{\mathbf{H}}$ is perpendicular to $A_{\mathbf{H}}$. The position vector (or the radius vector), $\boldsymbol{\rho}$, of the helix generated by point O has a magnitude equal to distance $O_{\mathbf{H}}O$ and is directed along $O_{\mathbf{H}}O$. Let $O_{\mathbf{H}r}$ be the projection of $O_{\mathbf{H}}$ on P_r and e the distance $O_{\mathbf{H}r}O$.

The following equations can now be derived [3]:

$$\boldsymbol{\omega} = \frac{V_c\sqrt{1 - \sin^2\eta\cos^2\theta}}{\rho}\{\cos\theta, \quad 0, \quad -\sin\theta\}_{XYZ} \tag{1}$$

$$p = (2\pi/\omega)V_T = \frac{2\pi\rho\sin\eta\cos\theta}{\sqrt{1 - \sin^2\eta\cos^2\theta}} \tag{2}$$

$$e = \rho_y = \frac{\rho\sin\eta\sin\theta}{\sqrt{1 - \sin^2\eta\cos^2\theta}} = \frac{V_T\tan\theta}{\omega} \tag{3}$$

where θ is angle between A_H and A_{Hr}, p is the pitch of helix H, and e is the offset e measured along the tool rake face between and the TCSL and A_{Hr}.

If it is assumed that the chip is in steady state helical motion as a rigid body after leaving the TCSL, every helix on the chip must have the same $\mathbf{V}_T$ and $\boldsymbol{\omega}$ (note that we have adopted the rotational viewpoint here). Applying the conditions of constancy of V_T, ω, and θ to equation (3), it follows that the magnitude of e must be the same for every point on the TCSL. The following are some significant implications of this simple analytical observation:

(i) The TCSL must be a straight-line segment collinear with axis X. Thus, whereas Nakayama had *assumed* the TCSL to be a straight-line segment, we have *proved* that it must be so. Further, the only requirement for this observation to be valid is that the chip should be circular helical. This leads to the following interesting question: Will the TCSL continue to be straight even when the rake plane contains some chip formers? (Note that, we often obtain circular helical chips even when the tool rake plane has chip former features.)

(ii) The straight TCSL must be parallel to the projection of the chip helix axis on the rake plane.

(iii)The distance e between this projection and the TCSL can be calculated by using equation (3).

(iv) The helical pitch p can be calculated by using equation (2).

Consider now another right handed set of Cartesian axes, X_V, Y_V, and Z_V centered at any point on the TCSL such that Y_V is directed along $\mathbf{V}_c$, and Z_V is normal to the rake plane ($Z_V \equiv Z$). The equation for transformation between the co-ordinate systems XYZ and $X_VY_VZ_V$ is available in [3]. Note that the radius vector $\boldsymbol{\rho}$ of total curvature at point O lies on the plane X_VOZ_V ($y_V = 0$ at point O_H). An expression for $\boldsymbol{\rho}$ can be determined in system XYZ by first determining it first in system $X_VY_VZ_V$ (by applying well-known principles concerning the geometry of a helix) and then transforming the result expression into the XYZ system. Thus

$$\boldsymbol{\rho} = \frac{\rho}{\sqrt{1 - \sin^2\eta\cos^2\theta}}\{-\sin\theta, 0, -\cos\eta\cos\theta\}_{X_vY_vZ_v} \tag{4a}$$

$$= \frac{\rho}{\sqrt{1 - \sin^2\eta\cos^2\theta}}\{-\cos\eta\sin\theta, \sin\eta\sin\theta, -\cos\eta\cos\theta\}_{XYZ} \tag{4b}$$

It is now easy to show that the total curvature, κ, of helix **H** is given by

$$\kappa = \frac{1 - \sin^2\eta\cos^2\theta}{\rho} \qquad (5)$$

Consider now the question of how one could logically define ρ_u and ρ_s in the context of a generalized (3-D) helical chip. Clearly, any definition set we choose must be consistent with the classical notions of up-curl and side-curl, i.e. the definitions should be plausible when applied to the cases of pure up-curl and pure side-curl (see section 2 for descriptions of these *pure* states of chip curl).

Following the above arguments, in [3], six equally plausible hypotheses concerning the definition sets were identified and the corresponding expressions for ρ_u and ρ_s were derived. Interestingly, it turned out that each of the hypotheses led to a distinct set of expressions. Thus, it became necessary to find further criteria to constrain the selection of the generalized definitions of ρ_u and ρ_s. This search led to the following three additional criteria:

- <u>Criterion 1</u>: $|\rho_u| = \rho$ *and* $|\rho_s| = \infty$ when $\eta=0$ and $\theta = 0$ (the case of pure up-curl).
- <u>Criterion 2</u>: $|\rho_u| = \infty$ *and* $|\rho_s| = \rho$ when $\theta = -90^o$ or 90^o (the case of pure side-curl).
- <u>Criterion 3</u>: No third radius component, ρ_3, that complements ρ_u and ρ_s should exist, <u>or</u>, *if* it exists, *then* the magnitude of ρ_3 should be equal to zero if the definition is derived from the viewpoint of chip rotation or, if the viewpoint adopted is one of chip curl, ρ_3 should have infinite magnitude
- <u>Criterion 4</u>: *If* the hypothesis defines ρ_u and ρ_s from the viewpoint of rotation, *then* $\sqrt{\rho_u^2 + \rho_s^2}$ must be equal to ρ. *If* the hypothesis defines ρ_u and ρ_s from the viewpoint of curvature, *then* $\sqrt{\frac{1}{\rho_u^2} + \frac{1}{\rho_s^2}}$ must be equal to κ (the magnitude of the total curvature, $\boldsymbol{\kappa}$, of the chip helix).

When the expressions for ρ_u and ρ_s developed from the six hypothetical definition sets were then tested against the four criteria described above, it was found that only one hypothesis had satisfied all the criteria. Hence it was concluded that the following are the logical definitions ρ_u and ρ_s in the context of the generalized (3D) helical chip:

- The radius of chip curl, ρ_u, is the radius of curvature of the chip helix at the TCSL in the direction normal to the rake plane, and is given by

$$\rho_u = \frac{1}{\kappa_z} = \frac{1}{\kappa_{z_v}} = \frac{\rho}{\cos\eta\cos\theta\sqrt{1 - \sin^2\eta\cos^2\theta}} \qquad (6).$$

- The radius of side-cur, ρ_s, is the radius of the remaining curvature in a direction parallel to the rake plane, and is given by

$$\rho_s = \frac{1}{\sqrt{\kappa_x^2 + \kappa_y^2}} = \frac{1}{\sqrt{\kappa_{x_v}^2 + \kappa_{y_v}^2}} = \frac{\rho}{\sin\theta\sqrt{1 - \sin^2\eta\cos^2\theta}} \qquad (7)$$

where κ_x, κ_y, and κ_z, are the curvatures, at point O (i.e. at the TCSL), of the projections of the chip helix **H** on planes Y_vZ_v, Z_vX_v, and X_vY_v respectively.

Note that the above analysis is derived from the viewpoint of chip-curl which is in contrast to the rotational viewpoint adopted by Nakayama [1, 2]. Therefore, it is useful to reexamine the new analysis from the view point of chip rotation.

Combining equation 3 with equation 7, it can be shown that

$$\omega_z = \frac{V_c}{\rho_s} \quad (8)$$

Likewise, combining equation 3 with equation 6, it is observed that

$$\omega_x = \frac{V_c}{\rho_u \cos\eta} \quad (9)$$

Now we may obtain expressions for θ and ρ by suitably combining and rearranging equations 8 and 9. Thus,

$$\tan\theta = \omega_z / \omega_x = (\rho_u \cos\eta) / \rho_s \quad (10)$$

$$\rho = \sqrt{\frac{1 - \sin^2\eta \cos^2\theta}{\frac{1}{(\rho_u \cos\eta)^2} + \frac{1}{\rho_s^{\,2}}}} \quad (11)$$

4. A Critique of Nakayama's Analysis [1, 2]

With the hindsight provided by the analysis developed in section 3, we are now in a position to critically review Nakayama's Analysis [1, 2].

Firstly, Nakayama had arrived at the same equations for pitch p and offset e as equations 2 and 3 respectively developed in the new analysis.

Secondly, equations 8 and 7 are identical to Nakayama's equations N1 and N6 respectively (see Table 1). This means that, whatever the arguments for or against the approach adopted by Nakayama, his analysis had correctly identified the chip's rotational velocity (ω_z) about the axis normal to the rake rake plane and, hence, the expression for ρ_s. This agreement has been possible because both Nakayama's analysis and the analysis presented in section 2 refer to the rake plane (plane XY) while defining ρ_s.

However, the same is not true with Nakayama's equation for ρ_u. Note that whereas the cosθ term is in the numerator in the right hand side of equation 6, it is in the denominator in equation N5. Thus, Nakayama's analysis leads to a greater and greater error in the estimation of ρ_u as the actual cutting situation deviates more and more from the state of pure up-curl (recall that θ = 0 in the state of pure up-curl).

Further, equation N2 of Nakayama's for the rotational velocity component ω_x is not in agreement with equation 9 developed in section 3.

The main problem with Nakayama's analysis is that, while defining ρ_u, he chooses the plane YZ that is normal to the TCSL whereas Hypothesis 3 chooses the plane passing through chip velocity vector $\mathbf{V}_c$ at the TCSL (both planes are of course normal to the rake plane and equally plausible in the context of pure up-curl)when viewed).

Other problems (of lesser consequence) associated with Nakayama's analysis have been discussed in [3].

4. EXTENSION OF THE NEW ANALYSIS TO COVER THE ENTIRE CHIP FACE

The analysis presented in section 3 had focused on the general helix **H** passing through point O**.** That analysis was extended in [4] so as to predict the geometry of the entire chip face (the screw surface). It was recognized that the chip face could be modeled of as a collection of helices ranging from $\mathbf{H}_0$ (the outer helix) to $\mathbf{H}_1$ (the inner helix). The extended analysis was conducted by repositioning the XYZ system to be centered at point O_0. Further, helix **H** was located with reference to the outer helix $\mathbf{H}_0$ by specifying a distance parameter *l* defined as the linear distance (along the TCSL) of point O from point O_0. This approach led to the following three equations

$$\rho = \rho_0 \frac{\sin\eta_0 \sqrt{1 - \sin^2\eta\cos^2\theta}}{\sin\eta\sqrt{1 - \sin^2\eta_0\cos^2\theta}} \tag{12}$$

$$\cot\eta = \cot\eta_0 - \frac{l\sin\theta\sqrt{1 - \sin^2\eta_0\cos^2\theta}}{\rho_0 \sin\eta_0} \tag{13}$$

$$V_c = V_{c0}\sin\eta_0 / \sin\eta \tag{14}$$

where ρ_0, η_0, and V_{c0} are the magnitudes of ρ, η,, and V_c respectively at point at point O_0.

Equation (13) determines η explicitly for an arbitrary point of the TCSL specified by a certain value of *l*. Substituting is value of η into equation (11), we can immediately determine ρ. Finally, applying equation (13), we can determine V_c. Thus,

- *once the basic parameter set,* (ρ, η, θ)*, and* V_c *at any point on the TCSL have been determined, the corresponding parameters at any other point specified by the *value of l on the TCSL are automatically determined.* Likewise, every helix on the chip surface is automatically determined once any one of the helices has been determined.

Note that η_0 and θ can, in principle, take up any value in the range -180^o to +180^o. However, a more detailed examination of the chip forms across this full range shows that they get repeated in different quadrants. Further, since negative up-curl is rarely found in practice, we may impose the condition that ρ_u cannot be negative at any point on the chip face. Following these arguments, two major conclusions were made in [4]:

- *The the parameter space* ($-90^o \leq \eta_0 \leq 90^o$, $-90^o \leq \theta \leq 90^o$) *encompasses all possible chip forms*.
- *The maximum permissible magnitude,* $|\eta_0|_{max}$*, of* η_0 *can be determined as*

$$|\eta_0|_{max} = \cot^{-1}\frac{|\sin\theta|}{\sqrt{\left(\frac{\rho_0}{l_1\sin\theta}\right)^2 - 1}} \tag{15}$$

According to (14), (15) *the parameter space (*η_0*,* θ*) of the possible chips becomes increasingly restricted with increasing relative chip width,* $|l_1|/\rho_0$*, along the TCSL*

5. CHIP-IN-HAND ANALYSIS

In [4], the analysis presented in section 3 was extended to show that it is possible to determine every other geometric parameter associated with the chip face from four easily measurable length dimensions of the chip-in-hand: (i) the outer radius, ρ_0; (ii) the inner radius, ρ_1; (iii) the magnitude of the pitch, |p|; and (iv) the magnitude of the 'axial width' (the axial displacement between the outer and inner helices) , $|h_1|$.

The following summarizes the unambiguous procedure developed in [4]:

1. Determine the magnitude of θ from the measured magnitudes of ρ_0, $|p|$ and $|h_1|$ by numerically solving the following equation

$$k_1 - \frac{\tan|\theta|}{\rho_0}\left(|h_1| + \frac{\alpha|p|}{2\pi}\right) = 0 \qquad (16)$$

where

$$\alpha = \cos^{-1}\frac{1 - k_1 k_2}{\sqrt{1 - 2k_1 k_2 + k_1^2}} \qquad (17)$$

(where the $\cos^{-1}$ value is evaluated in the range $[0, \pi/2]$) and

$$k_1 = k_2 - \sqrt{\left(\frac{\rho_1}{\rho_0}\right)^2 - \left(\frac{p\tan\theta}{2\pi\rho_0}\right)^2} \qquad (18)$$

where

$$k_2 = \sqrt{1 - \left(\frac{p\tan\theta}{2\pi\rho_0}\right)^2} \qquad (19)$$

2. From the measured magnitudes of ρ_0 and $|p|$ and the magnitude of θ determined in step 1, calculate the η_0 from the following equation

$$|\eta_0| = \sin^{-1}\frac{1}{\cos\theta\sqrt{1 + \left(\frac{2\pi\rho_0}{p}\right)^2}} \qquad (20)$$

3. Place the chip-in-hand on a horizontal surface, view it from the top, note the relative configuration of the chip face and the chip underside, and determine the signs of θ and η_0 by applying the special chart presented for this purpose in [4].
4. Determine whatever chip geometry parameter you are interested in by substituting the measured value of ρ_0 and the signed values of θ and η_0 in the appropriate equations developed in sections 3 and 5.

6. A NEW MAP OF CHIP FORMS

One of the major contributions of Nakayama was the pictorial map he had developed illustrating the different forms of 3-D chips that appear at different combinations of ρ, η, and θ [2]. This map has been an inspiration for many subsequent works on chip control.

However, the chip forms illustrated in Nakayama's map were derived from his analysis of the generalized (3D) helical chip. Section 4 has shown that this analysis is questionable and needs to be replaced with the analysis presented in sections 3 and 5 above. Hence, it is necessary to reformulate Nakayama's map in the light of our new analysis.

Figure ? shows the new map obtained from computer simulations based on the analyses developed in sections 3 and 5. It may be noted that, unlike Nakayama's map which had utilized three axes—one each for ρ, η, and θ, the new map utilizes only two axes—one for η as in Nakayama's map, and one for the parameter $\tan^{-1}(\rho_u/\rho_s)$ characterizing the curl-ratio ρ_u/ρ_s. This simplification has been prompted by the following notions:

(i) We need to be able to represent the specific helical form of the chip face. This form mainly depends on the curl ratio ρ_u/ρ_s (but not the absolute magnitudes of each of the curl components) and η. The curl ratio however is more elegantly characterized by using the term $\tan^{-1}(\rho_u/\rho_s)$ that avoids the problem arising when either ρ_u or ρ_s reaches infinity.

(ii) We also need to capture the orientation of the chip relative to the tool rake surface. This orientation depends mainly on θ and η. However, we know $\theta = \tan^{-1}\left(\frac{\rho_u}{\rho_s}\cos\eta\right)$. Hence, the effect of θ has already been implicitly captured by choosing the curl-ratio and η as the axes of our map.

CONCLUSION

A recent new analysis [3, 4] of the geometry of helical chips obtained in cutting with plane rake faced tools has been summarized.

In stark contrast to Nakayama's analysis [1, 2], which had attempted a 2D→3D analysis based on notions of chip rotation, the new analysis has adopted a 3D→2D approach based on notions of chip-curl while being consistent with the corresponding notions of chip rotation.

In contrast to Nakayama who had *assumed* that the tool-chip separation line is straight, the new analysis has *proved* this to be true.

The new analysis has led to equations 6 and 7 for determining the up-curl radii and side-curl radii from a given set of basic parameters related to the generalized (3D) helical chip. It is seen that, while equation 7 agrees with the corresponding equation developed by Nakayama for the side-curl radius, equation 6 differs significantly from equation N5 arrived at by Nakayama. The reason for this disagreement lies in the manner Nakayama had formulated the rotational motion associated with up-curl. Amongst the infinity of planes normal to the tool rake face, Nakayama had chosen the plane normal to the TCSL for defining up-curl. The new analysis has shown that this choice leads to results in disagreement with the total curvature of chip helix. The correct choice of the plane is the plane containing the chip velocity vector $\mathbf{V}_c$.

The new analysis has demonstrated that every basic parameter concerning every helix on the chip face is fully determined once the basic parameter set associated with one of the helices is known.

A practical benefit accorded by the new analysis is a new method for using the chip-in-hand to gain insights into the chip-in-process (at least in the case of lightly obstructed chips) and hence, possibly, into the chip formation process itself. In particular, it has been shown that every geometric parameter of interest associated with the face of the chip-in-process can be derived solely from four simple measurements conducted on the chip-in-hand. This finding is significant because these four parameters are in addition to the traditionally utilized measurements of chip length, thickness, and width to obtain insight into the chip formation process.

One aspect of Nakayama's work that has inspired many subsequent workers is the map of chip forms developed by him in terms of the radius of up-curl, radius of side-curl, and the chip flow angle specified with reference to the TCSL. However, the details of this map need now to be corrected following the discovery of errors in his formulation of up-curl. Figure ? shows the modified (and much simpler) mapdeveloped following the new geometric analysis of the generalized helical chip.

One limitation of the new analysis is that it is applicable to lightly obstructed chips where a significant proportion of practical chips are strongly obstructed (i.e chips that have undergone significant plastic deformation, owing to encounters with external obstacles, after passing over the TCSL. Further reserach is required to extend the new analysis summarized in this paper to the case of strongly obstructed chips.

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